

Improving Beam Pattern Estimation for Future Radio Telescopes

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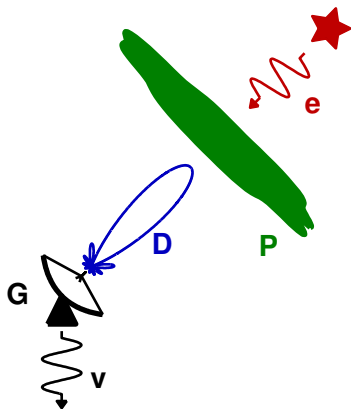
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Cape Town, South Africa

- 1 Motivation — Why do we need beam pattern models?
- 2 Beam Pattern Models
 - Analytic Pattern Basis Functions
 - Characteristic Basis Function Patterns
- 3 Novel Beamforming Approach

Outline

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Calibrating for antenna radiation pattern



RIME relates distant radio source signal to signal measured in telescope

$$\mathbf{v} = \mathbf{GDPe}$$

Signal transformations along propagation path (RHS factors) are calibrated to determine \mathbf{e} from \mathbf{v}

Goal of this study:

- Address calibration of \mathbf{D} through providing efficient radiation pattern models

Requirements for efficient pattern model

Pattern model in form of summation of basis functions¹

$$\mathbf{F}(l, m) = \sum_i^K \mathbf{x}_i \circ \mathbf{f}_i(l, m)$$

- model parameters = weighting coefficients $\{\mathbf{x}_i\}_{i=1}^K$
(full-polarisation, time and frequency dependent)
- trade-off: accuracy vs number of terms
- **suitable basis functions** $\mathbf{f}_i(l, m)$ that minimises number of terms

In search of the optimal set $\{\mathbf{f}_i\}_{i=1}^K \dots$

¹J. E. Noordam. "Measuring Station Beamshapes as a function of time and frequency and in full polarization". In: *3GC-II Workshop*. Albufeira (Portugal), Sept. 2011.

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Definition of analytic pattern functions

Based on a series solution for reflector antenna patterns²

$$F(\theta, \phi) = \sum_{n=0}^N \sum_{m=0}^M (C_{n,m} \cos n\phi + D_{n,m} \sin n\phi) \frac{J_q(ka \sin \theta)}{ka \sin \theta}$$

with $q = n + 2m + 1$.

Aperture distribution

- Azimuth: Trigonometric Fourier series
- Radial: Jacobi polynomials

Far-field

- Azimuth: Trigonometric Fourier series
- Elevation: Bessel + rational functions

²V. Galindo-Israel and R. Mittra. "A new series representation for the radiation integral with application to reflector antennas". In: *IEEE Trans. Antennas Propag.* 25.5 (Sept. 1977), pp. 631–641.

Calculating model coefficients

Typical (original) application:

- 1 Known primary pattern
- 2 Calculate current distribution on reflector (or aperture field distribution)
- 3 Calculate model coefficients (integral formula)
- 4 Calculate secondary pattern

Current application:

- 1 Secondary pattern known at few points
- 2 Estimate model coefficients (?)
- 3 Calculate secondary pattern

Estimating coefficients I: Point-matching

Point-matching to determine $K = (2N + 1)(M + 1)$ coefficients

- Number of basis functions
- = number of coefficients
- = **number of measurements**

$$\begin{bmatrix} f_1(\theta_1, \phi_1) & f_2(\theta_1, \phi_1) & \cdots & f_K(\theta_1, \phi_1) \\ f_1(\theta_2, \phi_2) & f_2(\theta_2, \phi_2) & \cdots & f_K(\theta_2, \phi_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(\theta_K, \phi_K) & f_2(\theta_K, \phi_K) & \cdots & f_K(\theta_K, \phi_K) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_K \end{bmatrix}$$

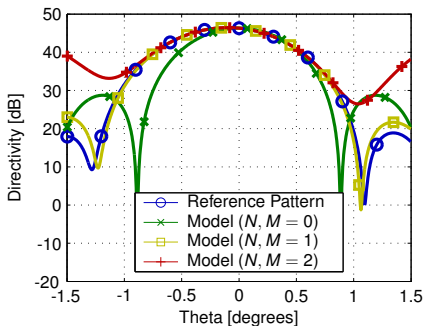
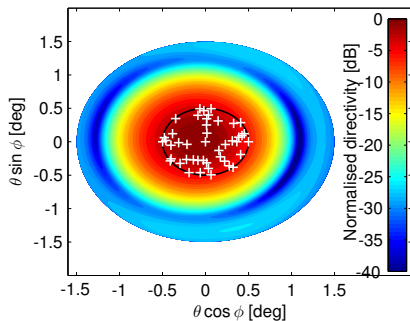
$$\mathbf{Ax} = \mathbf{V}$$

Sampling directions (θ_i, ϕ_i) selected to minimise $\kappa(\mathbf{A})$

Estimating coefficients I: Point-matching

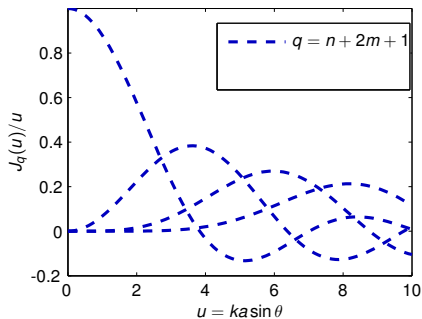
Analytic model for reference pattern

- MeerKAT + horn feed (small feed / subr position error)
- Reference pattern sampled within $\theta \leq 0.5^\circ$

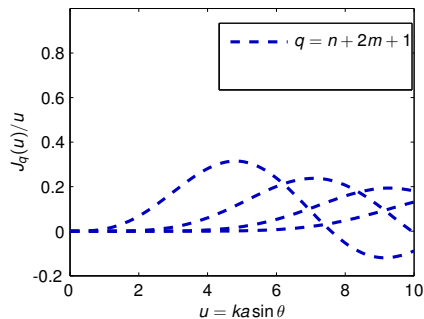


Increasing degrees of freedom

Order of Bessel function: $q = n + 2m + 1$ vs $q = m + 1$



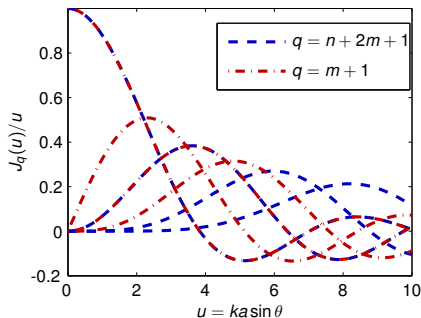
$n = 0, m = 0, 1, 2, 3$



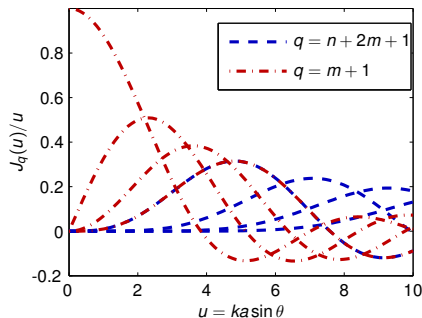
$n = 3, m = 0, 1, 2, 3$

Increasing degrees of freedom

Order of Bessel function: $q = n + 2m + 1$ vs $q = m + 1$



$n = 0, m = 0, 1, 2, 3$



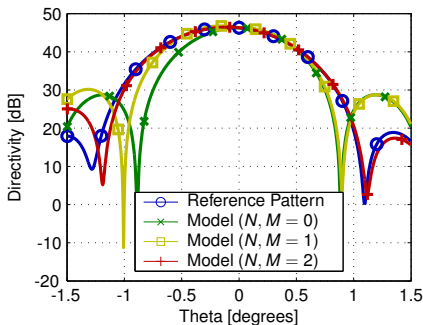
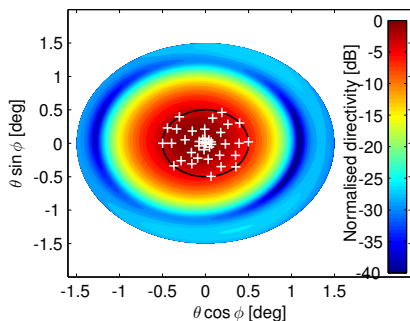
$n = 3, m = 0, 1, 2, 3$

$q = m + 1$ adds DoFs within certain angular region

Estimating coefficients I: Point-matching

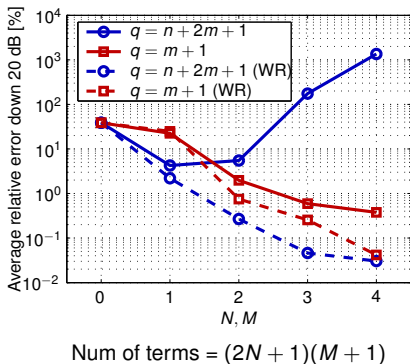
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- MeerKAT + horn feed (small feed / subr position error)
- Reference pattern sampled within $\theta \leq 0.5^\circ$
- $q = m + 1$



Estimating coefficients I: Point-matching

Compare model accuracy to “best” solution obtained through weighted residual approach



- DoFs added using $q = m + 1$ allows higher accuracy for point-matching approach

Estimating coefficients II: Constrained solution

Approximate pattern generally known on dense grid over large angular region (**ideally expected pattern**)

- Direction-by-direction measurements / accurate simulation models
- Use to calculate approximate coefficients through weighted residual method: $\mathbf{A}_W \mathbf{x}_0 = \mathbf{V}_W$

In practice estimate \mathbf{x} for **unknown reference pattern** from only a few measurements \mathbf{V}_R

- Performed often to compensate for pattern variations
- E.g. point-matching: $\mathbf{A}_R \mathbf{x} = \mathbf{V}_R$

If ideally expected pattern \approx unknown reference pattern

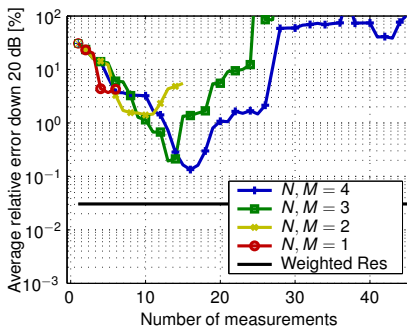
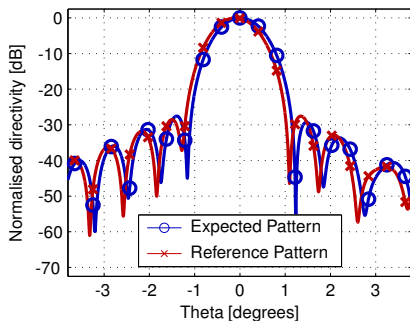
- $\mathbf{x}_0 \approx \mathbf{x}$

Estimating coefficients II: Constrained solution

Model secondary pattern for MeerKAT + horn feed

- Ideally expected pattern (\mathbf{x}_0) – no feed / subr error
- Unknown reference pattern (\mathbf{x}, \mathbf{V}_R) – small feed / subr error
- Minimise $\|\mathbf{x} - \mathbf{x}_0\|^2$ subject to $\mathbf{A}_R \mathbf{x} = \mathbf{V}_R$

$$\begin{bmatrix} \mathbf{I} & \mathbf{A}_R^H \\ \mathbf{A}_R & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{V}_R \end{bmatrix}$$

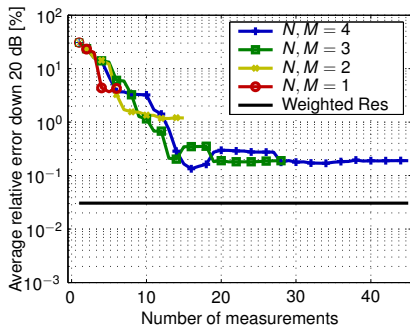


Estimating coefficients II: Constrained solution

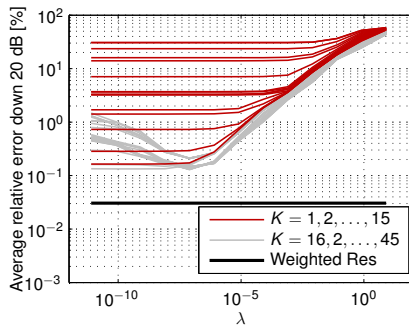
Model secondary pattern for MeerKAT + horn feed

- Ideally expected pattern (\mathbf{x}_0) – no feed / subr error
- Unknown reference pattern (\mathbf{x}, \mathbf{V}_R) – small feed / subr error
- Minimise cost function $\|\mathbf{A}_R \mathbf{x} - \mathbf{V}_R\|^2 + \lambda \|\mathbf{x} - \mathbf{x}_0\|^2$

$$\left(\mathbf{A}_R \mathbf{A}_R^H + \lambda \mathbf{I} \right) \mathbf{x} = \mathbf{A}_R^H \mathbf{V}_R + \lambda \mathbf{x}_0$$



$\lambda \approx 10^{-8}$



$N, M = 4$

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CBFP Method

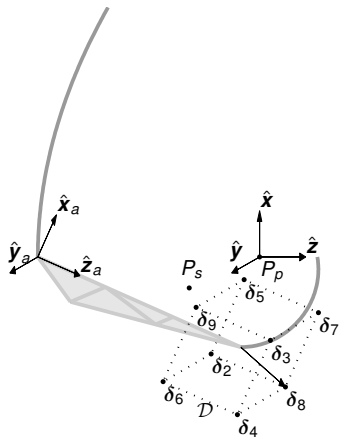
Use radiation patterns of various system configurations as basis functions³

- Primary CBFP is **ideally expected pattern** (perfect system)
 - Already contain much of system complexity (physics-based information)
 - Measurement / simulation
- Secondary CBFPs are perturbed patterns obtained after introducing certain errors in the system
 - Constructed to span space of patterns resulting from anticipated errors
 - Measurement (where possible) / simulation

³R. Maaskant et al. "Efficient Prediction of Array Element Patterns Using Physics-Based Expansions and a Single Far-Field Measurement". In: *IEEE Trans. Antennas Propag.* (scheduled for publication in 2012).

Application example: MeerKAT

Compensate for feed / subr displacement⁴



CBFP generation – sample radiation patterns on dense grid over wide angular region

- Primary: \mathbf{f}_1 obtained with $\delta = \mathbf{0}$
- Secondaries: \mathbf{f}_k obtained with $\delta = \delta_k$

⁴A. Young et al. “Accurate Beam Prediction Through Characteristic Basis Function Patterns for the MeerKAT/SKA Radio Telescope Antenna”. In review.

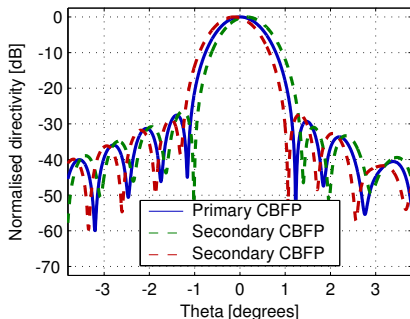
Application example: MeerKAT

Anticipated errors mainly result in pointing errors

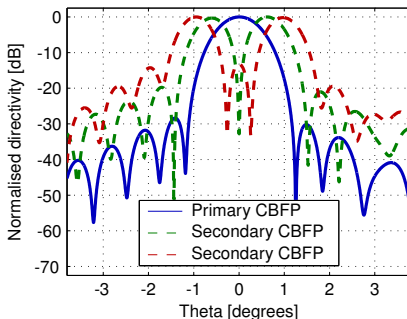
Adding large number of CBFPs result in redundancy (use SVD)

Choose first $L \leq K$ columns of \mathbf{U} as CBFPs ($\sigma_{1,2,\dots,L} \geq \sigma_\tau$) in

$$\mathbf{USV}^H = [\mathbf{f}_1 \mathbf{f}_2 \cdots \mathbf{f}_K]$$



Normalised CBFPs



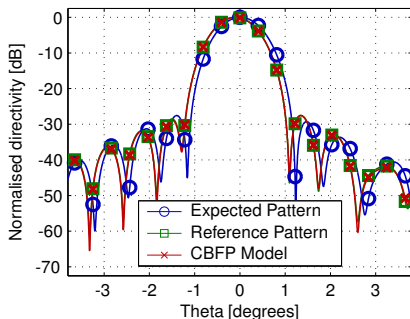
Normalised CBFPs after SVD

Results

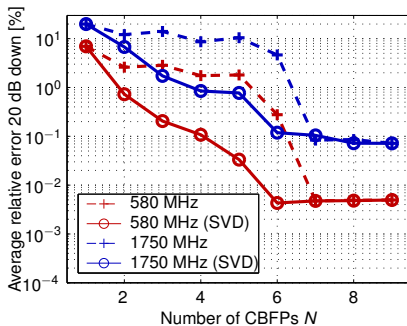
MeerKAT + horn feed

- Ideally expected pattern – no feed / subr error
- Unknown reference pattern – small feed / subr error

Use point-matching to obtain CBFP coefficients: $\mathbf{Ax} = \mathbf{V}$



9-term CBFP model for reference pattern



Model error with increasing CBFPs

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Beamforming for DAF-based systems

For Dense Array Feed based systems

- ✓ Enlarged FoV
- ✗ Increased calibration complexity

Beamforming techniques offer more flexibility to achieve desired performance

- Commonly considered for improving beam pattern performance (MaxSNR, reduced SLL, sensitivity ripple over FoV)
- Recently also focus on improving calibration efficiency through constrained beamforming⁵

⁵O. A. Lupikov, M. V. Ivashina, and O. M. Smirnov. “Reducing the complexity of the beam calibration models of phased-array radio telescopes”. In: *Proc. EuCAP*. Apr. 2011, pp. 930–933.

Constrained beamforming approach

Proposed novel approach uses Linearly Constrained Minimum Variance beamforming⁶

- Conform beam to physics-based analytic pattern function
- Ensure (**single-term**) model accuracy
- Least impact on instrument sensitivity

Modified jinc-function (2-parameter model)

$$F(\theta, \phi) = \frac{J_1(kaS \sin \theta)}{kaS \sin \theta} \exp(j\Psi \theta \cos(\phi - \phi_0))$$

- Parameters selected for desired pattern performance
- **Same parameter values** used for pattern model

⁶A. Young et al. "Improving the Calibration Efficiency of an Array Fed Reflector Antenna through Constrained Beamforming". In review. 2012.

Choosing model parameters

Proposed procedure to optimize parameter values

- 1 Create MaxDir beams over desired FoV
- 2 Derive initial model parameters from MaxDir beams
 - s_0 related to effective area
 - Ψ_0 related to phase pattern over main beam
- 3 Adjust parameter values (s, Ψ) to fine-tune performance

Parameter study for MeerKAT + Dense Array Feed

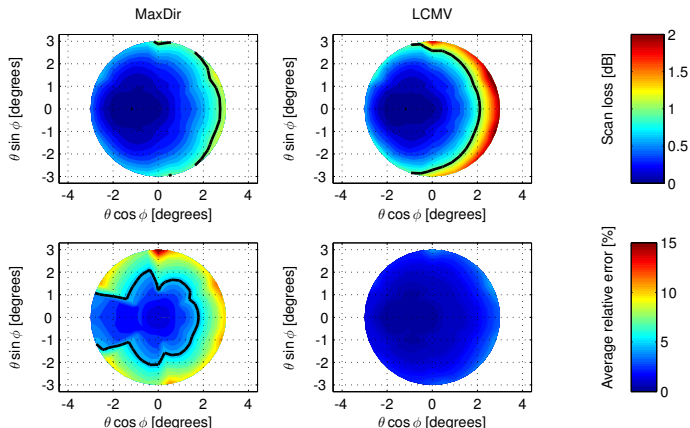
- $s \lesssim s_0$: Dir vs SLL, (accurate model over wide range)
- $\Psi \approx \Psi_0$: Best Dir, SLL, model accuracy

Designed LCMV beamformer with SLL < -17 dB over 3° FoV
(Comparison with MaxDir beamformer)

Results

MeerKAT + Dense Array Feed

- Average model error reduced by 66 %
- $\eta_{ap} > 70$ % FoV reduced by 18 % (SLL < -17 dB)



Summary

Provide **accurate** and **efficient** pattern calibration models

Analytic basis functions

- ✓ General model
- ✗ Large number of terms
 - Constrained solution

Characteristic basis function patterns

- ✓ Very few terms
 - Compensate for anticipated errors

Constrained beamforming

- Trade-off: calibration efficiency vs pattern performance

What next?

Analytic basis functions

- Other basis functions (shapelets, gaussian, etc.)

CBFPs

- Other anticipated errors (dish deformation, ground surface)
- Simultaneous compensation for various system errors

Hybrid analytic-characteristic approach?

Frequency dependence (OG freq ripple)

Experimental evaluation of various approaches

Thank You

Questions?

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