Improving Beam Pattern Estimation for Future Radio Telescopes

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Calibration and Imaging Workshop, Dec 2012
Cape Town, South Africa

Outline

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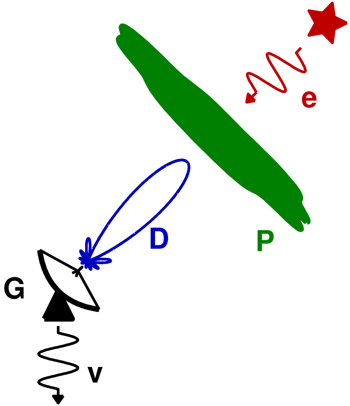
Calibrating for antenna radiation pattern



RIME relates distant radio source signal to signal measured in telescope

$$\mathbf{v} = \mathbf{G}\,\mathbf{D}\,\mathbf{P}\,\mathbf{e}$$

Signal transformations along propagation path (RHS factors) are calibrated to determine **e** from **v**

Goal of this study:

- Address calibration of **D** through providing efficient radiation pattern models

Requirements for efficient pattern model

Pattern model in form of summation of basis functions[1]

$$\mathbf{F}(l, m) = \sum_{i}^{K} \mathbf{x}_i \circ \mathbf{f}_i(l, m)$$

- model parameters = weighting coefficients $\{\mathbf{x}_i\}_{i=1}^{K}$ (full-polarisation, time and frequency dependent)
- trade-off: accuracy vs number of terms
- suitable basis functions $\mathbf{f}_i(l, m)$ that minimises number of terms

In search of the optimal set $\{\mathbf{f}_i\}_{i=1}^{K}$ ...

[1] J. E. Noordam. "Measuring Station Beamshapes as a function of time and frequency and in full polarization". In: *3GC-II Workshop*. Albufeira (Portugal), Sept. 2011.

Outline

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Definition of analytic pattern functions

Based on a series solution for reflector antenna patterns[2]

$$F(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=0}^{M} (C_{n,m} \cos n\phi + D_{n,m} \sin n\phi) \frac{J_q(ka \sin \theta)}{ka \sin \theta}$$

with $q = n + 2m + 1$.

Aperture distribution

- Azimuth: Trigonometric Fourier series
- Radial: Jacobi polynomials

Far-field

- Azimuth: Trigonometric Fourier series
- Elevation: Bessel + rational functions

[2] V. Galindo-Israel and R. Mittra. "A new series representation for the radiation integral with application to reflector antennas". In: *IEEE Trans. Antennas Propag.* 25.5 (Sept. 1977), pp. 631–641.

Calculating model coefficients

Typical (original) application:

1. Known primary pattern
2. Calculate current distribution on reflector (or aperture field distribution)
3. Calculate model coefficients (integral formula)
4. Calculate secondary pattern

Current application:

1. Secondary pattern known at few points
2. Estimate model coefficients (?)
3. Calculate secondary pattern

Estimating coefficients I: Point-matching

Point-matching to determine $K = (2N+1)(M+1)$ coefficients

- Number of basis functions
- = number of coefficients
- = number of measurements

$$\begin{bmatrix} f_1(\theta_1,\phi_1) & f_2(\theta_1,\phi_1) & \cdots & f_K(\theta_1,\phi_1) \\ f_1(\theta_2,\phi_2) & f_2(\theta_2,\phi_2) & \cdots & f_K(\theta_2,\phi_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(\theta_K,\phi_K) & f_2(\theta_K,\phi_K) & \cdots & f_K(\theta_K,\phi_K) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_K \end{bmatrix}$$

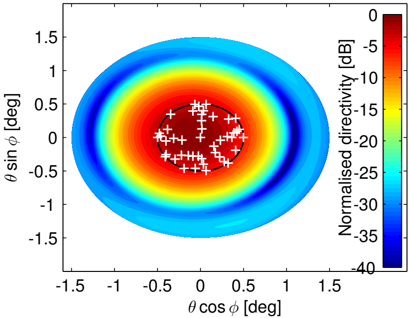
$$\mathbf{Ax} = \mathbf{V}$$

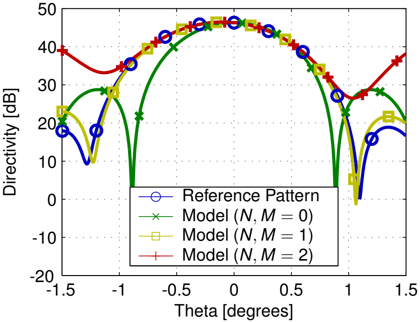
Sampling directions (θ_i, ϕ_i) selected to minimise $\kappa(\mathbf{A})$

Estimating coefficients I: Point-matching

Analytic model for reference pattern

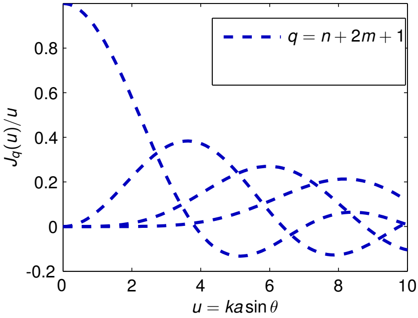
- MeerKAT + horn feed (small feed / subr position error)
- Reference pattern sampled within $\theta \leq 0.5^\circ$



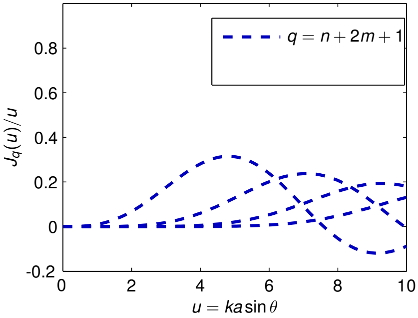


Increasing degrees of freedom

Order of Bessel function: $q = n + 2m + 1$ vs $q = m + 1$



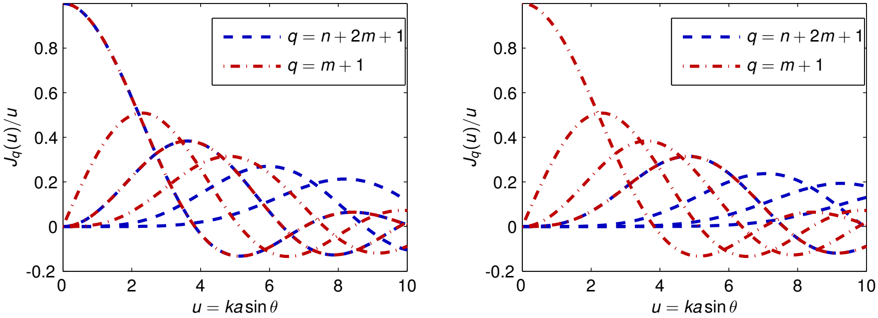
$n = 0, m = 0, 1, 2, 3$



$n = 3, m = 0, 1, 2, 3$

Increasing degrees of freedom

Order of Bessel function: $q = n + 2m + 1$ vs $q = m + 1$



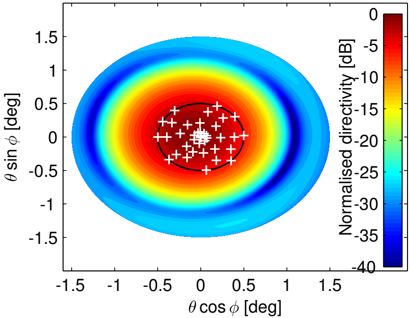
$n = 0, m = 0, 1, 2, 3$ $\quad$ $n = 3, m = 0, 1, 2, 3$

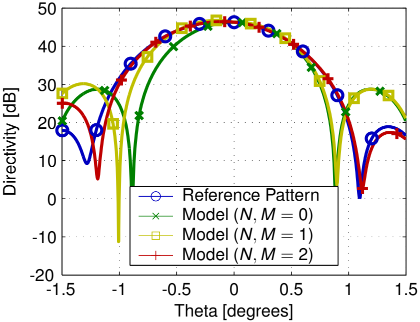
$q = m + 1$ adds DoFs within certain angular region

Estimating coefficients I: Point-matching

Analytic model for reference pattern

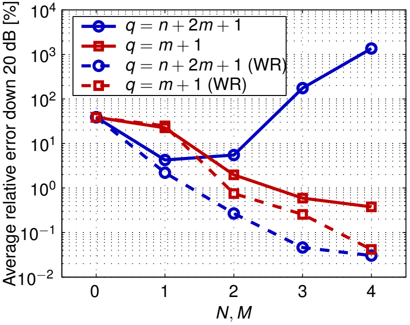
- MeerKAT + horn feed (small feed / subr position error)
- Reference pattern sampled within $\theta \leq 0.5^\circ$
- $q = m + 1$





Estimating coefficients I: Point-matching

Compare model accuracy to "best" solution obtained through weighted residual approach



Num of terms = $(2N + 1)(M + 1)$

- DoFs added using $q = m + 1$ allows higher accuracy for point-matching approach

Estimating coefficients II: Constrained solution

Approximate pattern generally known on dense grid over large angular region (ideally expected pattern)

- Direction-by-direction measurements / accurate simulation models
- Use to calculate approximate coefficients through weighted residual method: $\mathbf{A}_W \mathbf{x}_0 = \mathbf{V}_W$

In practice estimate $\mathbf{x}$ for unknown reference pattern from only a few measurements $\mathbf{V}_R$

- Performed often to compensate for pattern variations
- E.g. point-matching: $\mathbf{A}_R \mathbf{x} = \mathbf{V}_R$

If ideally expected pattern $\approx$ unknown reference pattern

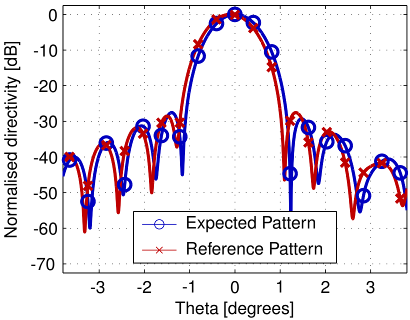
- $\mathbf{x}_0 \approx \mathbf{x}$

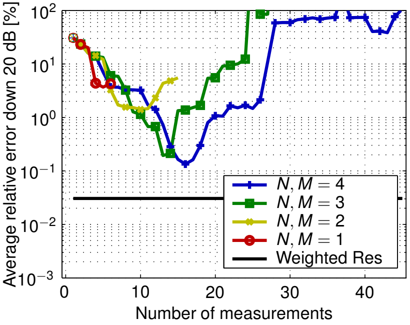
Estimating coefficients II: Constrained solution

Model secondary pattern for MeerKAT + horn feed

- Ideally expected pattern ($\mathbf{x}_0$) – no feed / subr error
- Unknown reference pattern ($\mathbf{x}, \mathbf{V}_R$) – small feed / subr error
- Minimise $\|\mathbf{x} - \mathbf{x}_0\|^2$ subject to $\mathbf{A}_R\mathbf{x} = \mathbf{V}_R$

$$\begin{bmatrix} \mathbf{I} & \mathbf{A}_R^H \\ \mathbf{A}_R & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{V}_R \end{bmatrix}$$



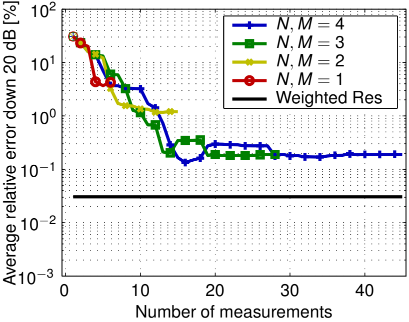


Estimating coefficients II: Constrained solution

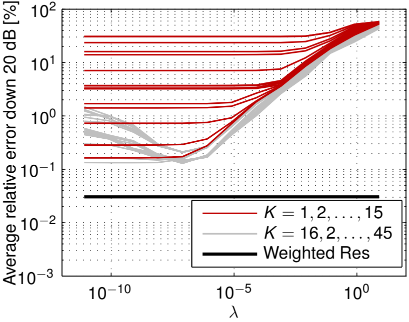
Model secondary pattern for MeerKAT + horn feed

- Ideally expected pattern ($\mathbf{x}_0$) – no feed / subr error
- Unknown reference pattern ($\mathbf{x}, \mathbf{V}_R$) – small feed / subr error
- Minimise cost function $\|\mathbf{A}_R\mathbf{x} - \mathbf{V}_R\|^2 + \lambda\|\mathbf{x} - \mathbf{x}_0\|^2$

$$\left(\mathbf{A}_R\mathbf{A}_R^H + \lambda\mathbf{I}\right)\mathbf{x} = \mathbf{A}_R^H\mathbf{V}_R + \lambda\mathbf{x}_0$$



$\lambda \approx 10^{-8}$



$N, M = 4$

Outline

CBFP Method

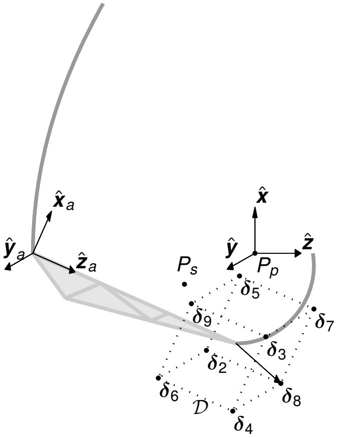
Use radiation patterns of various system configurations as basis functions[3]

- Primary CBFP is ideally expected pattern (perfect system)
  - Already contain much of system complexity (physics-based information)
  - Measurement / simulation
- Secondary CBFPs are perturbed patterns obtained after introducing certain errors in the system
  - Constructed to span space of patterns resulting from anticipated errors
  - Measurement (where possible) / simulation

[3] R. Maaskant et al. "Efficient Prediction of Array Element Patterns Using Physics-Based Expansions and a Single Far-Field Measurement". In: *IEEE Trans. Antennas Propag.* (scheduled for publication in 2012).

Application example: MeerKAT

Compensate for feed / subr displacement[4]



CBFP generation – sample radiation patterns on dense grid over wide angular region

- Primary: $\boldsymbol{f}_1$ obtained with $\delta = \mathbf{0}$
- Secondaries: $\boldsymbol{f}_k$ obtained with $\delta = \delta_k$

[4] A. Young et al. "Accurate Beam Prediction Through Characteristic Basis Function Patterns for the MeerKAT/SKA Radio Telescope Antenna". In review.

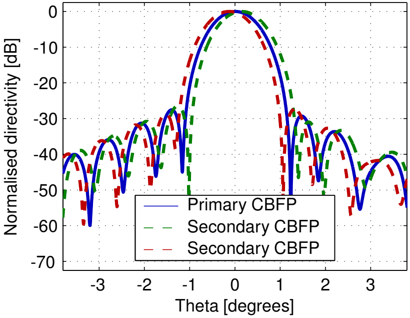
Application example: MeerKAT

Anticipated errors mainly result in pointing errors

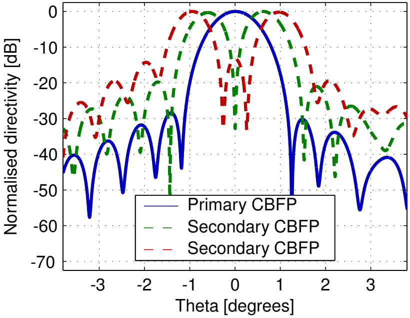
Adding large number of CBFPs result in redundancy (use SVD)

Choose first $L \leq K$ columns of $\mathbf{U}$ as CBFPs ($\sigma_{1,2,\ldots,L} \geq \sigma_\tau$) in

$$\mathbf{USV}^H = [\boldsymbol{f}_1\ \boldsymbol{f}_2\ \cdots\ \boldsymbol{f}_K]$$



Normalised CBFPs



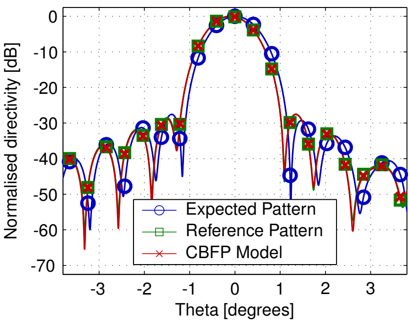
Normalised CBFPs after SVD

Results

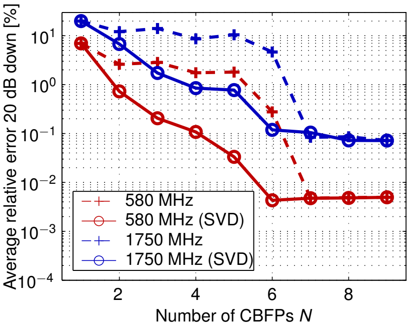
MeerKAT + horn feed

- Ideally expected pattern – no feed / subr error
- Unknown reference pattern – small feed / subr error

Use point-matching to obtain CBFP coefficients: $\mathbf{Ax} = \mathbf{V}$



9-term CBFP model for reference pattern



Model error with increasing CBFPs

Outline

Beamforming for DAF-based systems

For Dense Array Feed based systems

- ✔ Enlarged FoV
- ✖ Increased calibration complexity

Beamforming techniques offer more flexibility to achieve desired performance

- Commonly considered for improving beam pattern performance (MaxSNR, reduced SLL, sensitivity ripple over FoV)
- Recently also focus on improving calibration efficiency through constrained beamforming[5]

[5] O. A. Iupikov, M. V. Ivashina, and O. M. Smirnov. “Reducing the complexity of the beam calibration models of phased-array radio telescopes”. In: *Proc. EuCAP*. Apr. 2011, pp. 930–933.

Constrained beamforming approach

Proposed novel approach uses Linearly Constrained Minimum Variance beamforming[6]

- Conform beam to physics-based analytic pattern function
- Ensure (single-term) model accuracy
- Least impact on instrument sensitivity

Modified jinc-function (2-parameter model)

$$F(\theta, \phi) = \frac{J_1(kas \sin\theta)}{kas \sin\theta} \exp\left(j\Psi\theta \cos(\phi - \phi_0)\right)$$

- Parameters selected for desired pattern performance
- Same parameter values used for pattern model

[6] A. Young et al. “Improving the Calibration Efficiency of an Array Fed Reflector Antenna through Constrained Beamforming”. In review. 2012.

Choosing model parameters

Proposed procedure to optimize parameter values

1. Create MaxDir beams over desired FoV
2. Derive initial model parameters from MaxDir beams
    - s_0 related to effective area
    - Ψ_0 related to phase pattern over main beam
3. Adjust parameter values (s, Ψ) to fine-tune performance

Parameter study for MeerKAT + Dense Array Feed

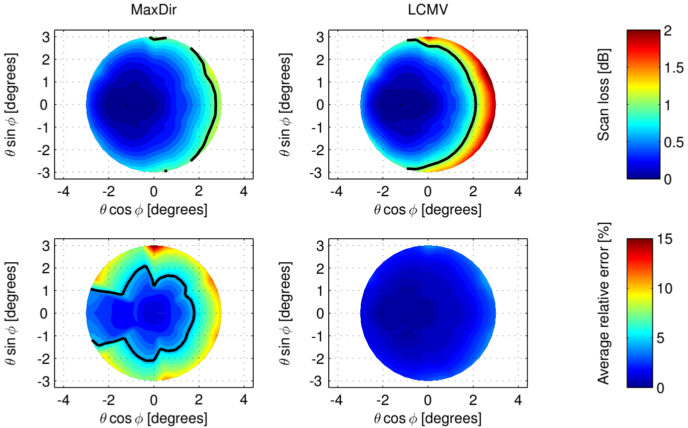
- $s \lesssim s_0$: Dir vs SLL, (accurate model over wide range)
- $\Psi \approx \Psi_0$: Best Dir, SLL, model accuracy

Designed LCMV beamformer with SLL < -17 dB over 3° FoV
(Comparison with MaxDir beamformer)

Results

MeerKAT + Dense Array Feed

- Average model error reduced by 66 %
- $\eta_{\text{ap}} > 70$ % FoV reduced by 18 % (SLL < -17 dB)



Summary

Provide accurate and efficient pattern calibration models

Analytic basis functions

- ✔ General model
- ✖ Large number of terms
  - Constrained solution

Characteristic basis function patterns

- ✔ Very few terms
- Compensate for anticipated errors

Constrained beamforming

- Trade-off: calibration efficiency vs pattern performance

What next?

Analytic basis functions

- Other basis functions (shapelets, gaussian, etc.)

CBFPs

- Other anticipated errors (dish deformation, ground surface)
- Simultaneous compensation for various system errors

Hybrid analytic-characteristic approach?

Frequency dependence (OG freq ripple)

Experimental evaluation of various approaches

Thank You

Questions?

This work was supported by

- South African Research Chairs Initiative of the Department of Science and Technology and National Research Foundation